

A new viscous fingering instability : the case of forced motions perpendicular to the horizontal interface of an immiscible liquid pair

B Roy and M H Engineer

Department of Physics, Bose Institute, Calcutta-700 009, India

Abstract : We report the discovery of a new, three dimensional instability in pairs of flowing immiscible liquids. A travelling ultrasonic wave sent along the axis of a vertical tube containing a pair of liquids sets up steady, circulating flows in both liquids. If the wave propagates from the less viscous member of the pair to the more viscous one the interface changes shape with the strength of the drive. First, a pronounced upward bulge develops. At a critical drive strength, a long finger of the less viscous fluid tunnels into the more viscous one. The phenomenon is as universal as the famous two dimensional viscous fingering instability discovered by Saffman and Taylor [*Proc. Roy. Soc. (London)* **A245** 312 (1958)].

Keywords : Interfacial instability, interfacial ripples, viscous fingering

PACS Nos. : 47.20.Gv, 47.35.+i, 47.55.Hd

1. Introduction

Interfacial waves generated by instabilities inherent in stratified flows of immiscible fluids have been discussed by several authors [1–3]. A striking example of the effects of fluid mechanical *non-linearity* on such waves was reported by Roy *et al* [4], and Chatterjee *et al* [5]. There the instability occurs when two immiscible fluids kept in a container are forced acoustically to move *parallel* to their horizontal resting interface. In the present paper we report an even more striking manifestation of non-linearity when stratified liquids are acoustically driven *perpendicular* to their resting interface.

2. Experimental details

A cylindrical glass tube, of inner diameter 3.42 cm and length 17 cm, was held with its axis kept vertical. A gold plated, X-cut Quartz transducer (Valpey Fisher Division, USA),

mounted axially at the base of the tube, was made to oscillate at around 5.0 MHz, using a tunable RF oscillator. The oscillating quartz plate sets up an ultrasonic wave which propagates into the liquids in the glass tube; as is well-known, such waves generate hydrodynamic flows in liquids [6] *via* the so-called quartz wind [see also 4,5]. The upper end of the glass container could be kept open or fitted with an acoustic terminator—sometimes a polythene disc terminator of diameter 3.4 cm and length 0.6 cm was used, and sometimes a carefully machined bakelite cone of length 5 cm. The heavier, and less viscous, liquid was first poured into the tube; thereafter the remaining space was filled, with the lighter and more viscous liquid. Use of this method allowed us to easily establish the necessary flow pattern while avoiding turbulence.

Experiments are reported here for two different oil-water systems : (i) the less viscous and denser liquid was water and the more viscous one was a mixture of castor oil and chloroform. The latter's density could easily be made very close to that of water by changing the proportions of its constituents; (ii) the less viscous and denser member was a mixture of carbon tetrachloride and petroleum ether and water was the other member of the pair. In both systems, the interfacial tension was lowered to a value of about 2 dyne/cm by mixing a small amount (0.2 %) of Triton X-100 (Fluke Chemie) in the distilled water. A negligible amount of water soluble (but oil insoluble) dye was used for clear identification of the liquids and their interface.

Generally we work with density differences of the order of 0.005 gm/cc and arrange for the less viscous liquid to be in contact with the driving ultrasonic transducer. We have observed that the behaviour reported is insensitive to whether the less viscous fluid is the denser or the lighter member of the pair.

3. Observations and results

The measured values of the densities, viscosities, ultrasonic velocities and the interfacial tensions of the liquids used are given in Table 1. The motion of fine suspended particles, at

Table 1. Measured values of parameters for liquids used.

Liquids	Density gm/cc	Viscosity poise	Interfacial tension dyne/cm (between water/oil mix.)
Water	1.01813	0.00882	
Pet. ether- CCl ₄ mixture	1.02404	0.0044	2
Water	1.01813	0.00882	
Castor oil- chloroform mixture	1.01108	4.42	2

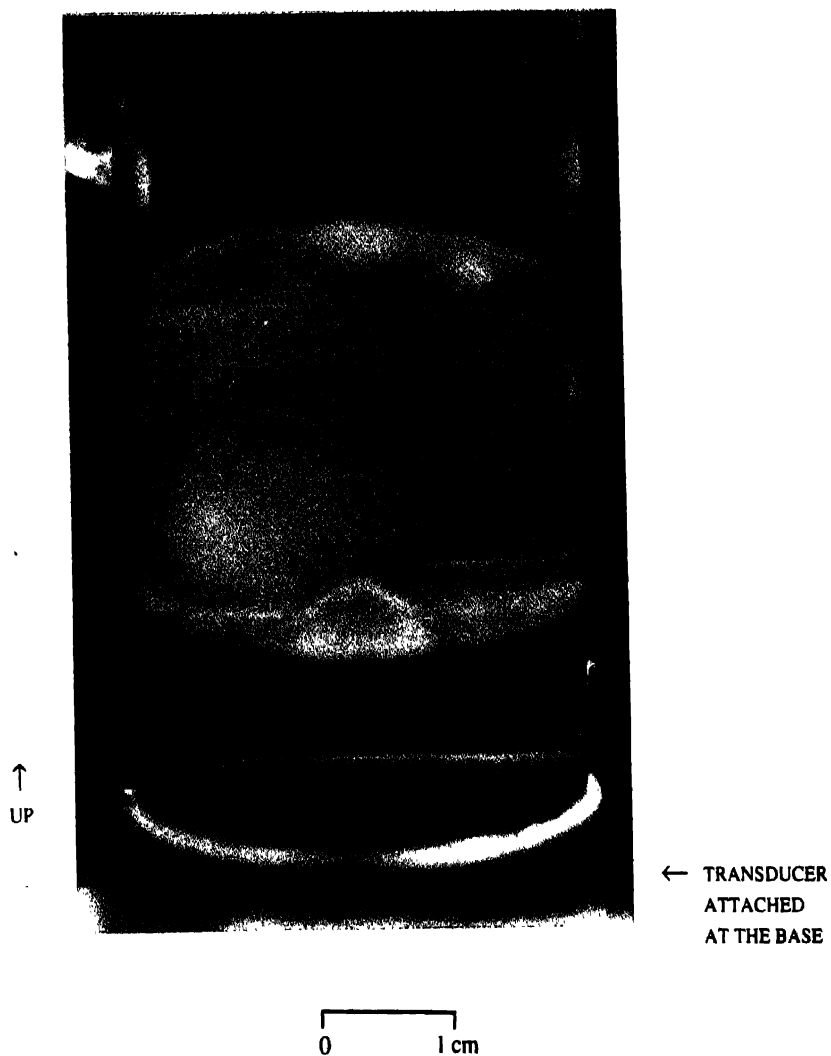


Figure 2. Viscous fingering instability with overall flow structure in water/castor oil-chloroform mixture system.



TRANSDUCER
ATTACHED
AT THE BASE

0 1 cm

Figure 3. Observed viscous fingering instability at the interface of water/castor oil-chloroform mixture

Plate III

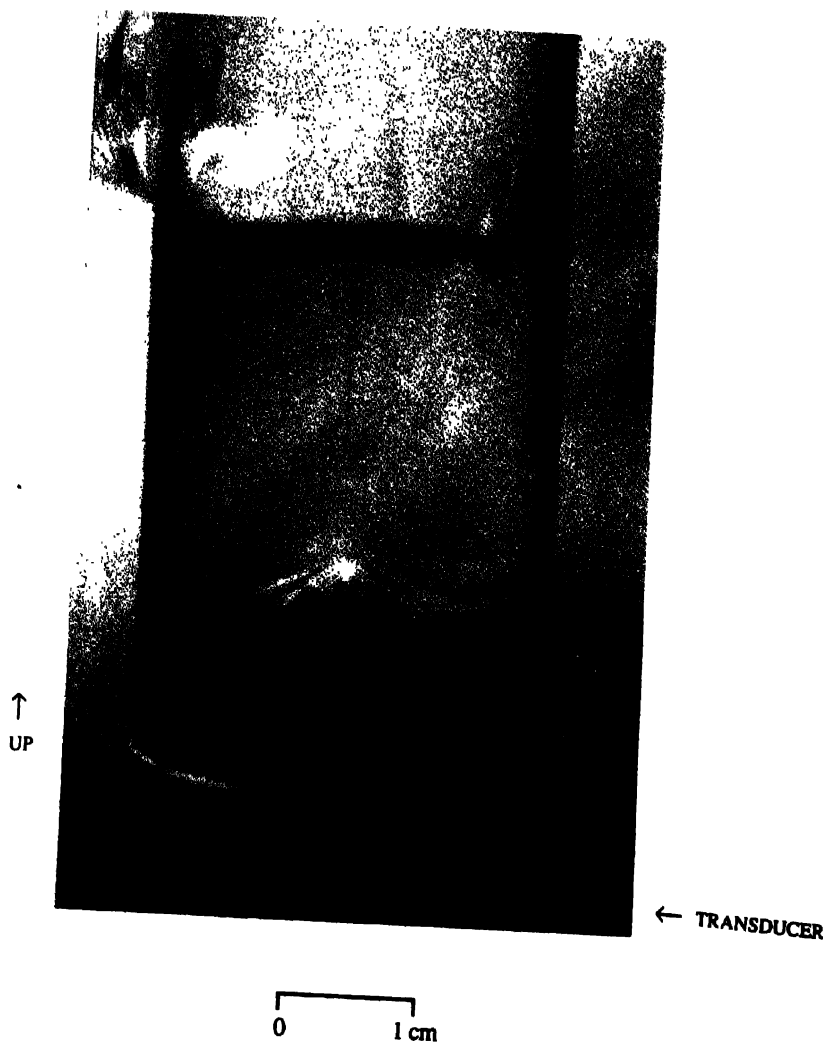


Figure 4. Observed viscous fingering instability in petroleum ether-carbon tetrachloride mixture/water system.

various mean flow velocities, is schematically shown in Figure 1(a-c) indicating that the steady velocity field is mostly circulatory on both sides of the interface. When the liquids

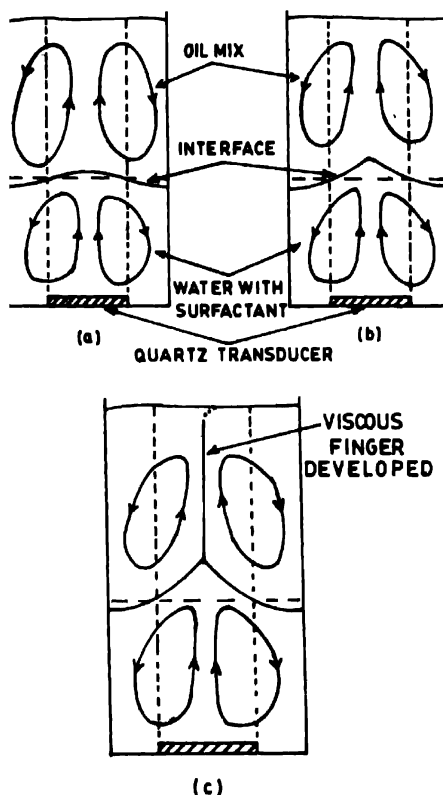


Figure 1. Schematic diagram of the fluid flow profile at different mean flow velocities.

are at rest, the interface is horizontal; the slight curvature at the glass walls arising from interfacial tension. The shape of the interface was observed to change markedly with change in driving acoustic power. In both oil-water systems, we examined the flow stability as a function of the strength of the driving acoustic power by observing the behaviour of the interface. The shapes remain unchanged for very small driving strengths, even though both fluids do flow. Eventually, beyond some critical flow strength, the less viscous liquid abruptly tunnels into the more viscous one and a transition takes place to a new state of steady flow. In the new state, the interface deforms strongly into an inverted funnel whose stem size is strongly system dependent. The tip of the stem breaks up into droplets of less viscous liquid at higher flow strengths.

- (i) The observed instability of the interface in case of water/castor oil-chloroform system is shown in Figure 2. The inverted funnel with thin stem can be seen at the centre. Figure 3 shows the overall nature of the flow as also the viscous finger.

- (ii) The instability observed in case of the water/petroleum ether- CCl_4 mixture system is shown in Figure 4. In this system the oil mixture is the less viscous liquid and it has been made denser than water. Since the flow was driven from oil to water it was expected that the less viscous oil mixture would finger into more viscous water, which indeed happened in our experiment. The shape of the interface in this second case differs in detail from that observed in (i). However, the fingering phenomenon is present though the finger is both thicker and unsteady. Occasionally droplets of oil break off from the tip of the finger and fall back into the oil. All these observations are yet to be explained theoretically.

4. Conclusions

Interfacial instabilities in the form of inverted funnels have been observed in both the oil/water systems studied. In the well-known viscous fingering phenomenon [7], the growing finger points from the less to the more viscous liquid; the tip of the inverted funnel has exactly the same property in our experiments *i.e.*, only less viscous liquids can tunnel into more viscous ones. Accordingly, the present study reveals that the viscous fingering instability is possible in fully three dimensional flows as well, contrary to the existing belief in the fluid mechanics community.

References

- [1] S Chandrasekhar *Hydrodynamic and Hydromagnetic Stability* (New York : Dover) (1961)
- [2] S A Thorpe *J. Fluid Mech.* **39** 25 (1969)
- [3] S A Thorpe *J. Geophys. Res.* **92** 5231 (1987)
- [4] B Roy, B K Chatterjee, M H Engineer and Pradip Roy *Physica A* **186** 250 (1992)
- [5] B K Chatterjee, M H Engineer, B Roy and Pradip Roy *J. Fluid Mech.* **248** 663 (1993)
- [6] J E Piercy and J Lamb *Proc. Roy. Soc. (London)* **A226** 43 (1954)
- [7] P G Saffman and S G Taylor *Proc. Roy. Soc. (London)* **A245** 312 (1958)